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TECHNICAL ARTICLE—PEER-REVIEWED

# Buckling Failure Analysis of Hydraulic Cylinder Rod on the Flap Institutions for Power Catwalk

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**Abstract** This paper analyzes the mechanical characteristics of the hydraulic cylinder rod and the causes of buckling failure. The structural buckling failure often happens to the hydraulic cylinder rod on the flap institutions during the power catwalk operation. The calculation formula of the buckling critical force of the hydraulic cylinder rod is derived by applying the modified boundary conditions. The formula of moment of inertia of different cross sections is deduced, and the calculation formula of the critical force of the hydraulic cylinder rod with defects is obtained by doing research on the rod with the cylindrical defects. Experiments on buckling of hydraulic cylinder rod were conducted on the basis of the simplified test specimen, and the buckling numerical simulation of rod was carried out by using the finite element software Workbench. Through the numerical simulation, we analyze how the radius, depth, and position of the defect to impact the buckling load. The experimental and simulation results show that the formula is accurate when calculating the buckling load of the defect-free hydraulic cylinder rod, and the variation of the buckling load with the cylinder defect parameters was also analyzed.

**Keywords** Hydraulic cylinder rod · Cylindrical defect · Buckling load · Experimental study · Numerical simulation

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## Introduction

Some components and part of the power catwalk are easy to have partial or overall buckling failure during the operation due to heavyweight and a large number of the pipe columns [1, 2]. Under consideration of the installation position, the high frequency of pushing pipe column, and the large thrust, the hydraulic cylinder rod on the flap institutions is much easier to result in buckling instability, which causes the buckling problems on it. These problems need to be further researched [3–5].

Buckling failure of engineering components has been studied by many researchers with different methods. Those include the experimental and numerical methods [6], the final failure response of damaged composite-stiffened panels in post-buckling regime under compressive load [7], inelastic buckling analysis [8], engineering approach and finite element analysis [9], the initial imperfection approach [10], Bubnov–Galerkin method [11], and the other kinds of research [12–14].

When calculating the stability of the compression rod, Euler formula is used to calculate the critical force of the compression rod [15, 16]. The force acting on the hydraulic cylinder rod can be simplified as axial pressure, its flexibility value is also greater than the limits, and hence it is suitable for Euler formula [17]. The result of the calculation has certain errors because of the simplicity assumption of the structure. Therefore, the more accurate boundary condition should be used for calculating its critical force. The rod end compression displacement is measured by the test method, and its critical force is re-determined in this paper. The formulas of moment of inertia and the buckling load of hydraulic cylinder rod with the cylindrical defect are also given in this paper. According to the analysis and experiment, the finite element model of the hydraulic cylinder rod is established. The buckling of the hydraulic cylinder rod with different defect parameters is analyzed using a numerical simulation method.

## Analysis on the Critical Force of Hydraulic Cylinder Rod on the Flap for Power Catwalk

### Working Conditions of Hydraulic Cylinder Rod on the Flap Institutions for Power Catwalk

As shown in Fig. 1, the hydraulic cylinder rod is mainly used to turn the pipe column in or out of the main body of power catwalk. The harsh working conditions, high working frequency, and unequal axial loads, which are caused by the inconsistent operation of multiple hydraulic cylinders in the field, cause the buckling failure of the cylinder rod. Therefore, it is necessary to determine the critical force of the hydraulic cylinder rod to ensure its stable operation [2] (Fig. 2).

### Equation of the Critical Force of the Compression Rod

The buckling problems of compression rod were studied by many scholars [18–20]. The flap cylinder rod can actually be simplified as a compression rod since the ratio of the length to its diameter is between 10 and 15. Based on the past research [3], as shown in Fig. 2, we simply assume the cylinder rod on the flap institutions as the compression rod and apply the Euler pressure rod bending formula. Consequently, the approximate differential equation of the critical deflection curve under the action of force $F$ is:

$$y'' + k^2 y = 0 \tag{Eq 1}$$

where $k^2 = \frac{F}{EI}$, $E$ is the elastic modulus of the material, Pa;$I$ is the cross-section moment of inertia of the pressure rod, $m^4$.

To solve this equation, the more accurate boundary conditions: $x = 0, y = 0; x = l - \lambda, y = 0$, are adopted. The critical pressure of the hydraulic cylinder rod should be:

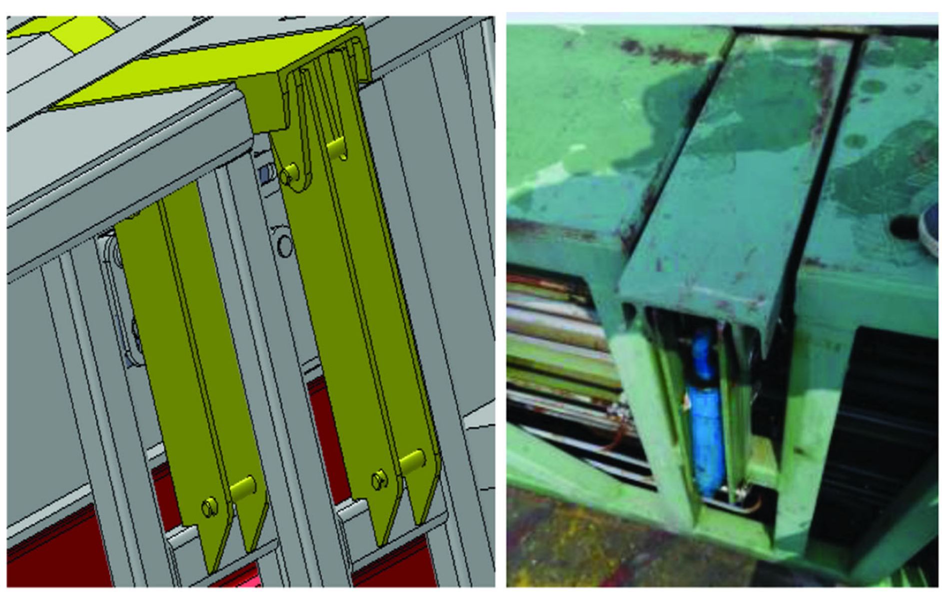
$$P_{cr} = \frac{\pi^2 EI}{l^2 \left(1 - \frac{\lambda}{l}\right)^2} \tag{Eq 2}$$

where $\lambda$ is the compression displacement of the hydraulic cylinder rod along the axial direction, and $l$ is the length of the rod.

According to the method of calculating the critical force of the piston rod [21], the coefficient $\mu$ is defined as the influence caused by installation and the guide of the hydraulic cylinder rod should also be taken into account. The final critical force is calculated as follows:

$$P_{cr} = \frac{\pi^2 EI}{\mu^2 l^2 \left(1 - \frac{\lambda}{l}\right)^2}. \tag{Eq 3}$$

The material of hydraulic cylinder is 42CrMo where the elastic modulus is 206 Gpa. The minimum cross-sectional moment of inertia of the hydraulic cylinder rod is chosen as



**(a) Flap institutions of the power catwalk** **(b) Hydraulic cylinder rod of the power catwalk**

**Fig. 1** Flap institutions and hydraulic cylinder rod of the power catwalk. (**a**) Flap institutions of the power catwalk. (**b**) Hydraulic cylinder rod of the power catwalk

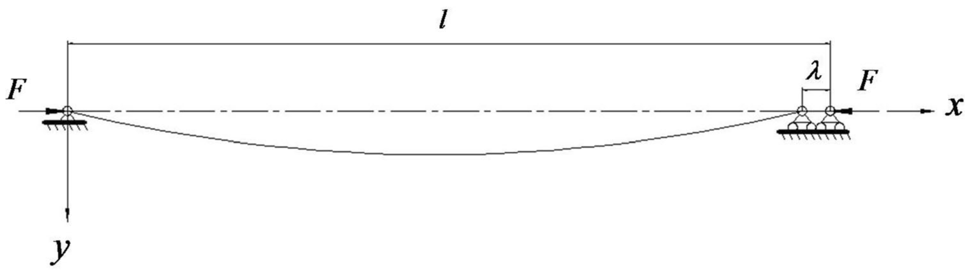



**Fig. 2** Calculation model of pressure rod while considering end face displacement

the moment of inertia $I$, $I = \frac{\pi}{64}d^4$, $d = 10\,\text{mm}$; $l$ is 120 mm which is defined as the distance between the bottom and the center of the pin hole in the upper end of the hydraulic cylinder rod. According to mechanical design manual [22] and installation situation, the coefficient $\mu$ is equal to 2. The $\lambda$ needs to be quantitatively solved before solving the differential equation of the hydraulic cylinder rod.

## Calculation of Buckling Critical Force of the Hydraulic Cylinder Rod with Cylindrical Defect

### Analysis of the Phenomenon of Hydraulic Cylinder Rod with Defect

According to the field application statistics, the occurrence of defect of the hydraulic cylinder rod is critical for rods buckling failure when it has been used for a period of time. Contrasting to the defect holes on the unbalanced hydraulic cylinder rod, it is found that there are two common types of defect which are similar to the circular and elliptical holes, as shown in Fig. 3. For this reason, the rod with cylindrical defect is chosen as the studying object. The influence of defect diameter, depth, and position on the critical force of hydraulic cylinder rod is analyzed.

Based on the elastic mechanics theory [23–25], the expression of stress $\sigma_z$ on the $x$ and $y$ plane (except for the hole) of the cylinder rod with cylindrical defect is:

$$\sigma_z = \sigma\left[1 + \frac{4-5\mu}{14-10\mu}\frac{r^3}{R^3} + \frac{9}{14-10\mu}\frac{r^5}{R^5}\right] \qquad \text{(Eq 4)}$$

where $\sigma$ (MPa) is the far field tensile stress; $\mu$ is Poisson's ratio; $R$ (mm) is the radius of the rod; $r$ (mm) is the corrosion hole radius.

The defect causes non-uniform changes in cross-sectional area of the hydraulic cylinder rod because of its non-uniformity. In addition to the strength checking, it is of great importance to prevent buckling failure. Due to the existence of the cylindrical defect, the buckling critical force of the hydraulic cylinder rod cannot be calculated by Eq 3, and hence a new method needs to be developed.

### Calculation of Moment of Inertia of Hydraulic Rod with Defect

The connecting section between the bottom end of the hydraulic cylinder rod on the flap institutions and the hydraulic cylinder liner along with the connection section of the upper flap shaft can be simplified into straight sections for the convenience of analysis. As shown in Fig. 4, the distance $L_0$ is the length from the bottom of the flap to the center of the connecting shaft hole.

The moment of inertia along y and z directions of hydraulic cylinder rod with cylindrical defect can be calculated by applying the knowledge of material mechanics [25]:

$$\begin{aligned} I_y &= \frac{\pi d^4}{64} - \frac{bh}{4}(d-h)^2 \\ I_z &= \frac{\pi d^4}{64} - \frac{hd^3}{12} \end{aligned} \qquad \text{(Eq 5)}$$

where $d$ (m) is the diameter of the cylinder, $h$ (m) is depth of the defect, and $b$ (m) is the width of the defect.

According to the position of the defect in simplified hydraulic cylinder rod straight section and the relevant geometric relations, b and h can be calculated as follows:

$$b = \begin{cases} 0 & (0 < x \le L - r \& L + r < x \le L_0) \\ 2\sqrt{r^2 - (x-L)^2} & (L - r < x \le L + r) \end{cases}$$
$$h = \begin{cases} 0 & (0 < x \le L - r \& L + r < x \le L_0) \\ t & (L - r < x \le L + r) \end{cases} \qquad \text{(Eq 6)}$$

Substituting Formula 6 into Formula 5, the moment of inertia along y and z directions of the simplified straight section of the cylinder rod with defect can be calculated as:

$$I_y = \begin{cases} \frac{\pi d^4}{64} & (0 < x \le L - r \& L + r < x \le L_0) \\ \frac{\pi d^4}{64} - \frac{2t\sqrt{r^2-(x-L)^2}}{4}(d-t)^2 & (L - r < x \le L + r) \end{cases}$$
$$I_z = \begin{cases} \frac{\pi d^4}{64} & (0 < x \le L - r \& L + r < x \le L_0) \\ \frac{\pi d^4}{64} - \frac{td^3}{12} & (L - r < x \le L + r) \end{cases} \qquad \text{(Eq 7)}$$

**Fig. 3** Defect in hydraulic cylinder rods after buckling

where $L$ (m) is the distance from the center of the defect to the bottom of the hydraulic cylinder rod, $r$ (m) is the defect radius, and $t$ (m) is the defect depth.

### The Calculation of Buckling Critical Force of the Hydraulic Cylinder Rod with Cylindrical Defect

Substituting $I_y$ which is calculated by substituting Eq 7 into Eq 3, the critical force corresponding to the different sections for the hydraulic cylinder rod with cylindrical defect can be calculated as follows:

$$P_{\mathrm{cr}} = \begin{cases} \dfrac{\pi^3 E d^4}{64 k^2 l^2 \left(1-\frac{\lambda}{l}\right)^2} & (0 < x \le L - r \,\&\, L + r < x \le L_0) \\ \dfrac{\pi^2 E}{k^2 l^2 \left(1-\frac{\lambda}{l}\right)^2}\left[\dfrac{\pi d^4}{64} - \dfrac{2t\sqrt{r^2-(x-L)^2}}{4}(d-t)^2\right] & (L - r < x \le L + r) \end{cases} \qquad \text{(Eq 8)}$$

Theoretically, the minimum value of Eq 8 is the critical force of the cylinder rod on the flap institutions.

$$P_{cr} = \frac{\pi^2 E}{k^2 l^2 \left(1-\frac{\lambda}{l}\right)^2}\left[\frac{\pi d^4}{64} - \frac{2tr}{4}(d-t)^2\right] \qquad \text{(Eq 9)}$$

The above equation shows that the radius and depth of cylindrical defect have an obvious influence on the critical force of the hydraulic cylinder rod of different sections. Due to the non-uniform radius of hydraulic cylinder rod on flap institutions with cylindrical defect, the basic calculation cannot yet be fully expressed by Eq 8. Consequently, the simplified numerical model analysis of the flapper hydraulic cylinder rod with cylindrical defect was carried out, and the influence law of the defect parameters on the stability was analyzed in the following study.

## Experimental Study on Buckling of Hydraulic Cylinder Rod

### Test Device

A test had been made on the microcomputer controlled electrohydraulic servo universal testing machine as shown in Fig. 5 to accurately calculate the buckling critical force of the hydraulic cylinder rod and determine the compression displacement of the buckling rod end. The test machine has a restraint device both up and down. Loading process is controlled by the software. The maximum load of the test machine is 200 KN. The relevant sensor can transmit the real-time data to the software terminal which can automatically collect the data.

### Test Specimen and Its Data Collection

The flap hydraulic cylinder rod test sample is clamped on both ends by V-block clamp, while the top is loaded in terms of fixture performance of the universal testing machine to study on the hydraulic cylinder rod's buckling load. The two-dimensional structure diagram is shown in Fig. 6. The effective length of the specimen is 120 mm which is equal to the distance that the flap hydraulic cylinder rod to the upper end of the central axis. The diameter is 10 mm equal to the minimum diameter of hydraulic cylinder center rod.

The test machine can provide force in the form of sine, linear, cosine, etc. We prepared four specimens to do the test, and the estimated buckling load is 17 KN according to Euro equation. The initial linear load we set for test machine is 5 KN, and the data acquisition is 0.2 KN.

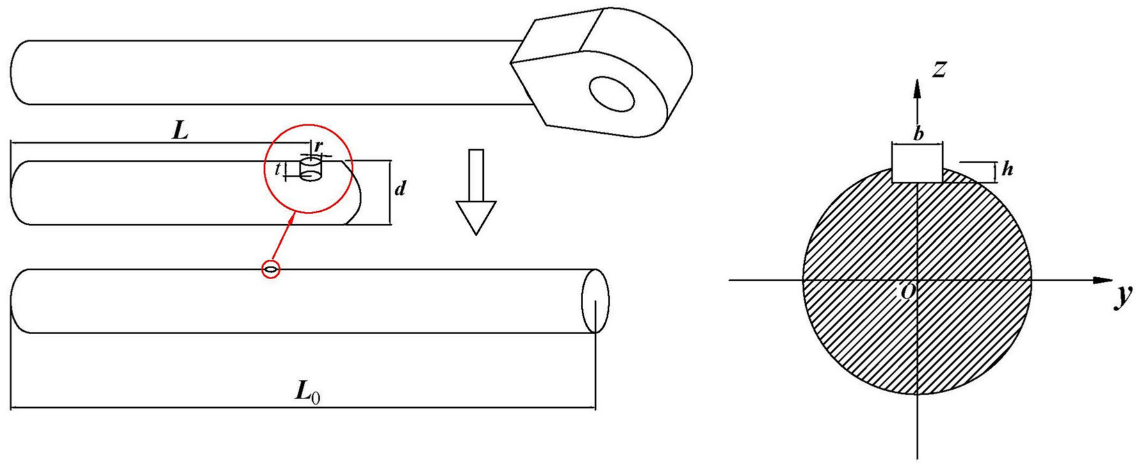



**Fig. 4** Structure diagram of hydraulic cylinder rod with defect

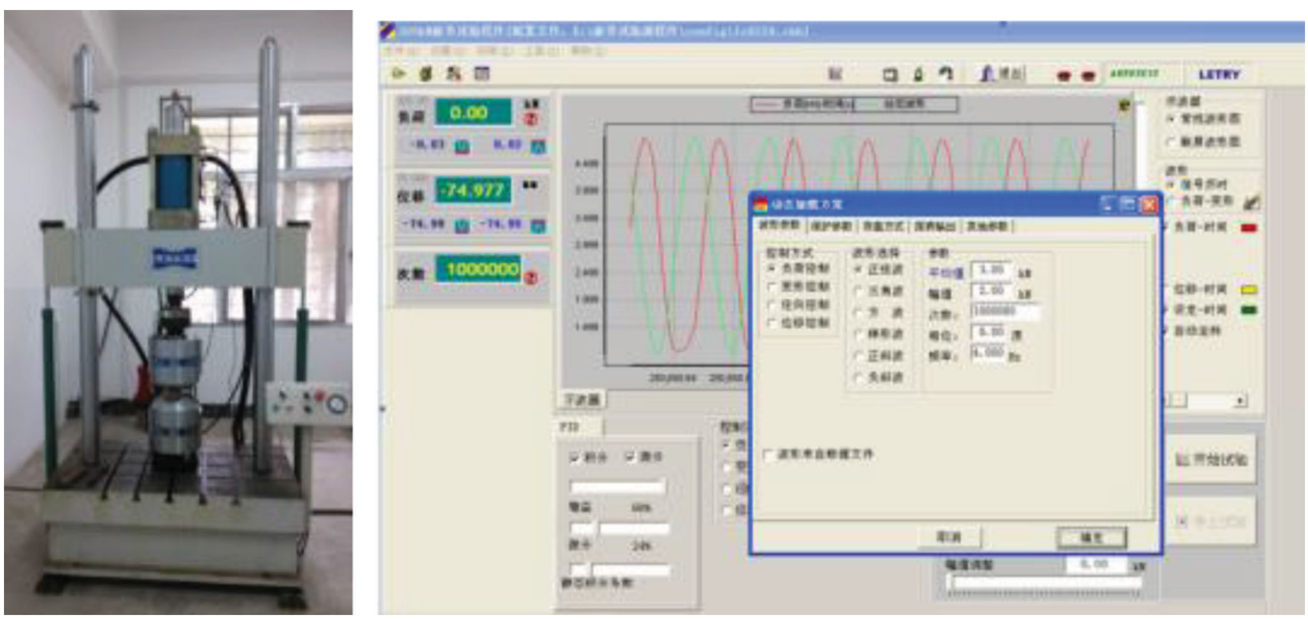



**Fig. 5** Microcomputer-controlled electrohydraulic servo universal testing machine

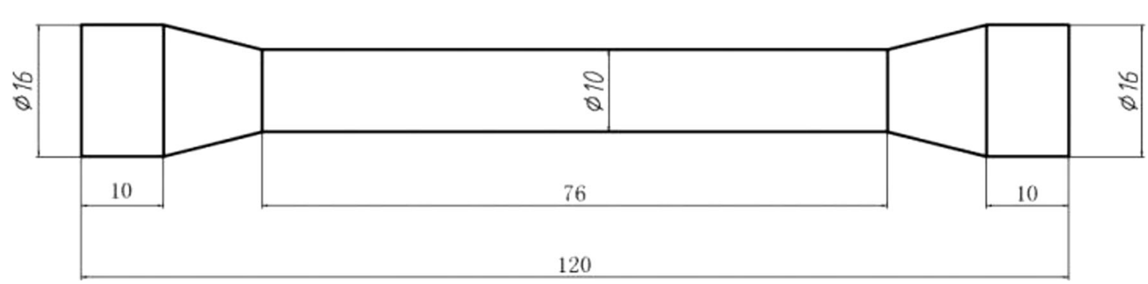



**Fig. 6** 2D figure of specimen

### Test Data Analysis

The relationship between F and ε as shown in Fig. 6 can be found through the collected data which had been processed by the analysis software. The strain of the specimen has increased dramatically when the load reaches 16.8 KN, 12.4 KN, 17.2 KN and 16.2 KN for specimens 1, 2, 3 and 4, respectively, as shown in Fig. 7. It can be determined from Fig. 6 that there is a branch point for respective specimen under the corresponding load which is considered as the buckling critical force. At the same time, we stop loading the test rod 2 because it has a large deformation after loading to 12.4 KN. The critical force of specimen 2 is small because it has defect after observing it, and there have obvious holes on it. The critical force of specimen 2 is out of comparison range in the subsequent non-defect hydraulic cylinder rod research.

The tested critical value for the hydraulic cylinder rod specimen can be calculated by averaging specimens 2, 3 and 4:

$$\bar{F}_{\mathrm{cr}} = \frac{16.8 + 17.2 + 16.2}{3} = 16.73\,\mathrm{kN} \quad \text{(Eq 10)}$$

At the same time, the collected data show that the compression displacement of the rod is less than 0.5 mm. We amplify it to 1 mm and plug it in Formula 3 to solve the calculation.

$$P_{\mathrm{cr}} = \frac{\pi^2 EI}{\mu^2 l^2 \left(1 - \frac{\lambda}{l}\right)^2} = \frac{3.14^2 \times \frac{3.14}{64} \times (0.01)^4 \times 206 \times 10^9}{2^2 \times 0.12^2 \times \left(1 - \frac{1}{120}\right)^2} = 17301\,\mathrm{N} \quad \text{(Eq 11)}$$

## Numerical Simulation Model

### Model Geometric Parameters

The three-dimensional model of the hydraulic cylinder rod imports the Workbench which is established before analysis. As shown in Fig. 8, the solid 186 unit is used, the rod is divided into 108,661 tetrahedral unit, the number of nodes is 157,190, and the mesh of the defective part is also refined.

### Material Parameters and Loading Solution

The material model is the ideal elastic–plastic model where the elastic modulus $E = 2.06 \times 10^5$ MPa, and Poisson's ratio $\mu = 0.3$. The material selected for the hydraulic cylinder rod is 42CrMo, and the yield strength is $\sigma_s = 930\,\mathrm{MPa}$.

When solving buckling problems, the buckling load is determined by the calculated load factor. The reached load factor is the cylinder rod's characteristic value, i.e., the buckling critical force. Therefore, the associated structural static and buckling analyses are carried out under the condition that the bottom surface of the flap cylinder is fixed and the upper shaft hole is free and the force along the axial direction is 1 N.

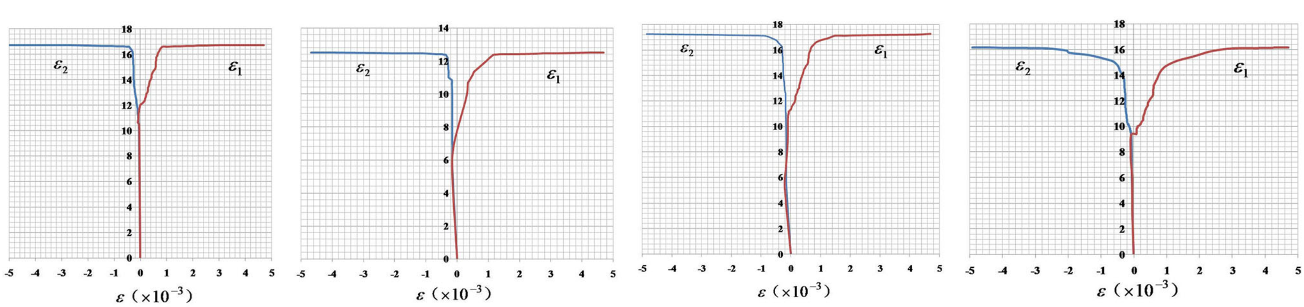
### Calculation Results of Defect-Free Hydraulic Cylinder Rods

When using ANSYS software for analysis, the linear buckling analysis is actually the study of eigenvalues. The theoretical limit load and the critical force of the rod are obtained by solving the eigenvalues combined with the added loads. The basic solving equation is:

$$[K + \eta S]\psi = 0, \quad \text{(Eq 12)}$$

where $K$ is the overall stiffness matrix for the hydraulic cylinder rod. $S$ is the stress-hardening matrix for the hydraulic cylinder rod. $\psi$ is buckling mode displacement array for the hydraulic cylinder rod. $\eta$ is eigenvalue.

The first-order buckling load of the defect-free hydraulic cylinder rod by finite element method is 17415 N, as shown in Fig. 9. Comparing it with the theoretical values, the error of theoretical results is 0.65% in the reference of




(a) Specimen1 (b) Specimen2 (c) Specimen3 (d) Specimen4

**Fig. 7** $F - \varepsilon$ relationship diagrams of specimens. (**a**) Specimen1. (**b**) Specimen2. (**c**) Specimen3. (**d**) Specimen4

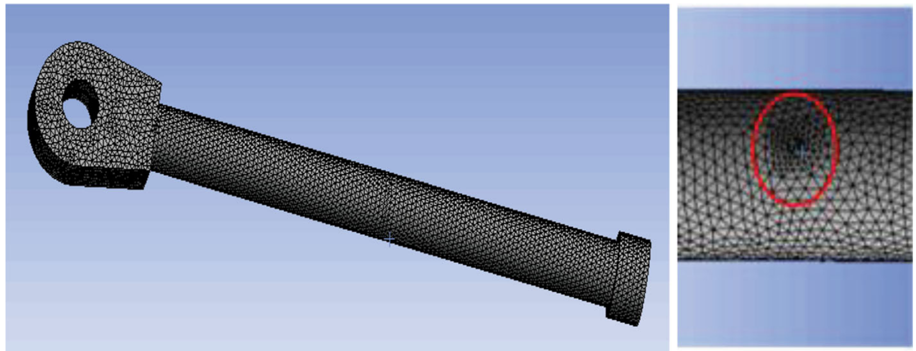


**Fig. 8** Grid partition graph and refinement graph of hydraulic cylinder rod

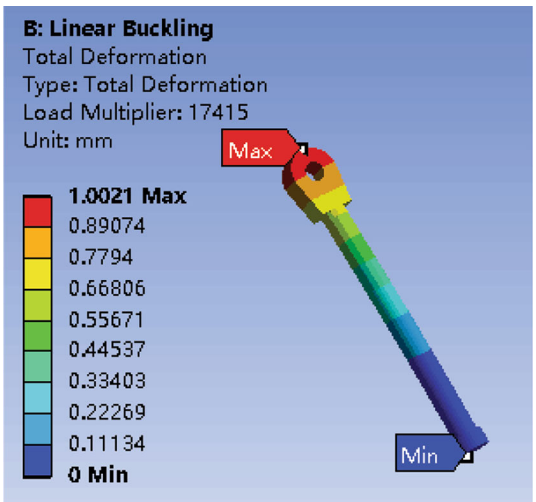



**Fig. 9** Buckling deformation diagram of hydraulic cylinder rod

the value calculated by finite element method. The experimental values are smaller than the other two values. The mean error of the test is 3.93%, and the maximum error (sample 4) is 6.98%. For the numerical simulation of the defective hydraulic cylinder rod, test specimen 2 shows that the effect of the defect on the buckling of the hydraulic cylinder rod is obvious, so the buckling of the defective hydraulic cylinder rod will be analyzed emphatically.

## Buckling Analysis of Hydraulic Cylinder Rod with Cylindrical Defect

The single-variable method is used to further study how the cylindrical defect impacts the bucking load of the defective hydraulic cylinder rods. We finally obtained the variation law of buckling load factor of flap hydraulic cylinder rod with different defects as shown in Fig. 10; the analyses include the defect radius from 0.5 to 2.5 mm, the defect depth from 1 to 5 mm and the defect distance from 10 to 115 mm.

### The Effect of Defect Depth on the Buckling Load of Hydraulic Cylinder Rod

Figure 10 shows the buckling load factor (buckling load) of the hydraulic cylinder rod increases with the increase in the cylindrical defect depth h. Furthermore, the increase in amplitude from 1 mm to 2 mm is sharp, but it decreases at a depth in the range of 3 mm to 5 mm. When the distance from the bottom of the hydraulic cylinder rod increases, the increased amplitude of the buckling load of the hydraulic cylinder rod is continued to decrease.

### Influence of Defect Position on Buckling Load of Hydraulic Cylinder

By comparing with a) through e) in Fig. 10, it can be seen that the influence of the defect position on the buckling load of the hydraulic cylinder rod for the catwalk on the institutions is basically the same, and the change of the value is obvious. As the distance from the center of the defect to the upper end of the rod increases, the buckling load increases and the increased amplitude decreases. The above findings suggest that the hardly checked part of the hydraulic cylinder rod should be adopted into important checking factors, especially the inner rod of the hydraulic cylinder in addition to the easily checked upper part of it, which is one of the routine inspections.

### Influence of Defect Radius Size on Buckling Load of Hydraulic Cylinder Rod

The above-mentioned studies mainly compare the effects of different depths and different position of defects on buckling load of the hydraulic cylinder rod. In addition to the comparisons of a) through e) diagrams of Fig. 10, the results of group $h = 3$ are extracted to form Fig. 11 for the comparison to find out the influence of different defect radius sizes on the stability of the hydraulic rod. Figure 11 shows that, when the distance between the defect position and the bottom of the hydraulic cylinder rod is within a

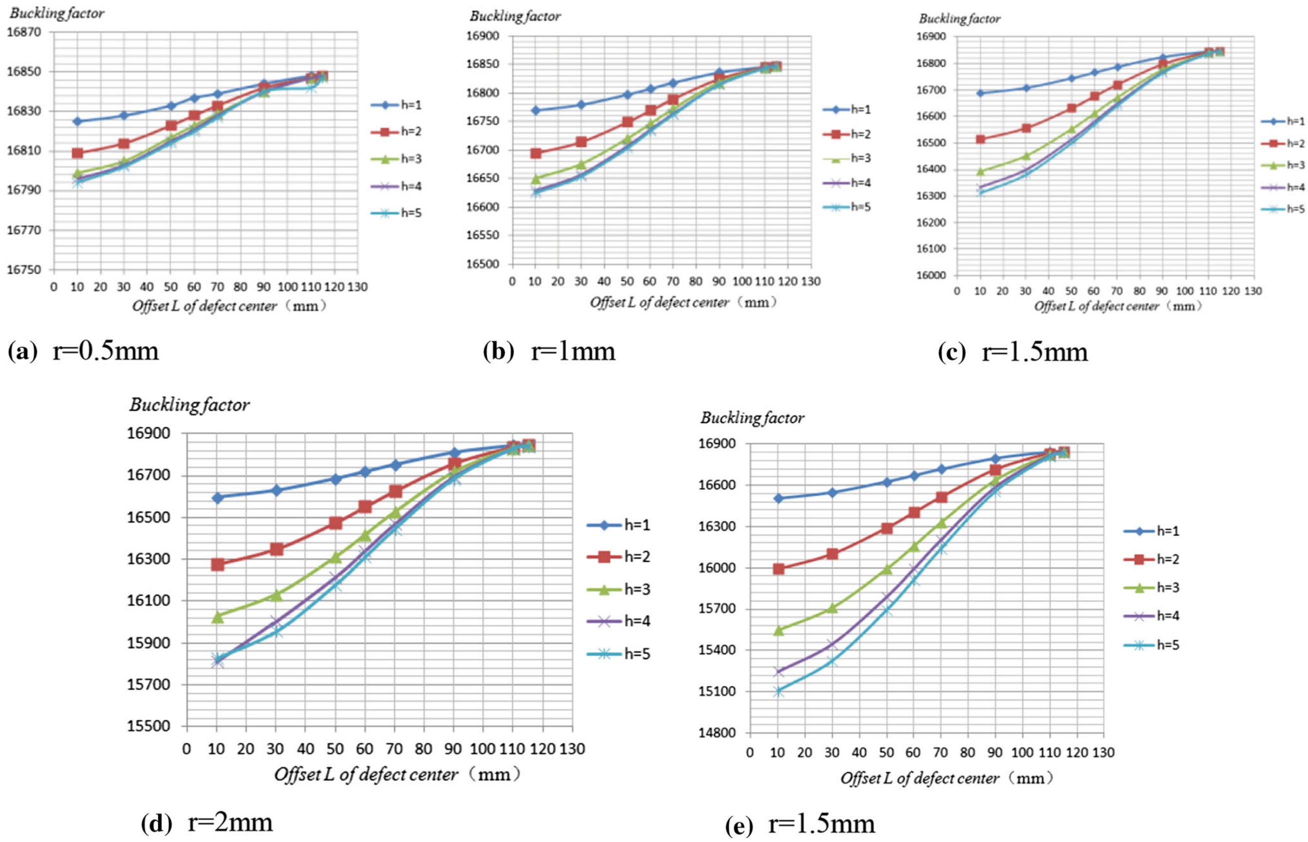



**Fig. 10** Change law of buckling factor. (**a**) $r$ = 0.5 mm, (**b**) $r$ = 1 mm, (**c**) $r$ = 1.5 mm, (**d**) $r$ = 2 mm, (**e**) $r$ = 1.5 mm

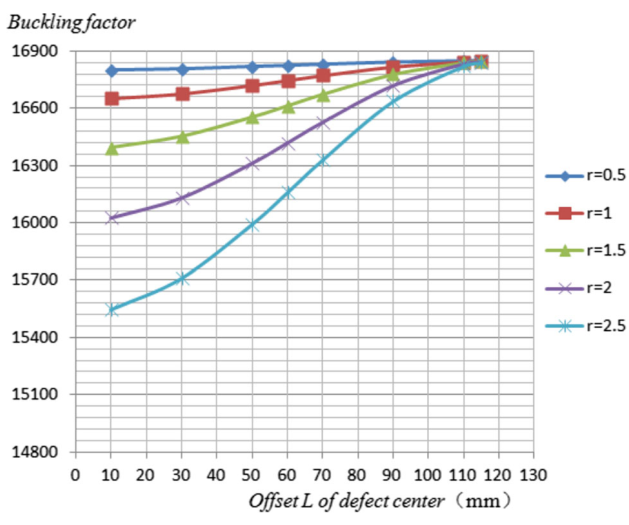



**Fig. 11** Buckling factor change with the radius of the defect. (**a**) Flap institutions of the power catwalk. (**b**) Hydraulic cylinder rod of the power catwalk

certain range, the influence of the diameter of the defect on the buckling load of the hydraulic cylinder rod is obvious. Meanwhile, the buckling load of the hydraulic cylinder rod is reduced, and the decreased amplitude is increased with the defect radius increase (Fig. 11).

## Conclusions

In this paper, the buckling critical force of hydraulic cylinder rod is analyzed and calculated by the formulas as presented, and it is further tested by experiments and numerical simulations base on the problem of buckling failure of hydraulic cylinder rod on the flap institutions for power catwalk used in oil/gas production. As a result of the study, the following observations can be made:

1. There is an obvious difference between the Euler formula of conventional compressive rod and the formula of the buckling critical force of defect-free hydraulic cylinder rod on the flap institutions under consideration of the boundary conditions: $x = 0, y = 0; x = l - \lambda, y = 0$, and the coefficient $\mu$. The results of experiment and numerical simulation both validate the correctness of the critical force formula of the non-defective cylinder hydraulic rod on the flap institutions that is deduced by us and presented in this paper.
2. The formula of buckling load of the hydraulic cylinder rod with cylindrical defects is deduced. By using this formula, the analysis shows that the depth and radius

of the defect have a great influence on the buckling load.

3. The $F - \varepsilon$ curve of the defect-free hydraulic cylinder rod has been obtained by applying the test. The theoretical buckling critical force of the defect-free hydraulic cylinder is calculated by the tested values. The buckling critical force measured by the test is compared with the calculated results and the simulation results. The maximum error between each other is less than 7%. It is also found that defects have a great influence on the hydraulic cylinder buckling critical force.
4. The numerical simulation results show that the buckling critical force is significantly reduced with the increase of the depth and radius of the defect. At the same time, it is found that the value of the buckling critical force is closely related to the position of the defect. The influence of the defect on the buckling force is greater when it is closer to the bottom of the hydraulic cylinder rod.
5. The research made in this paper indicates that, after a period of work, the hardly checked part, which is the end of the hydraulic cylinder rod should be adopted into important checking factors to avoid buckling failure to insure the safety of the hydraulic cylinder rod, especially the inner rod of the hydraulic cylinder in addition to the easily checked upper part of it, which should be one of the routine inspections.

In addition to the cylindrical defects, there are other types of defects. Therefore, we will further study how the micro-bending state, different defect shapes, and other parameters to impact the stability of the compression rod.

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